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Effects of a Uniform Gentle Slope on the Seabed Boundary Layer Beneath Obliquely Propagating Linear Waves Plus a Cross-Shore Current

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Abstract

A finite-difference model solving the simplified Reynolds-averaged equations, combined with a two-equation k-ε turbulence closure, is used to investigate the effects of a uniform gentle slope on turbulent wave boundary layers beneath obliquely propagating linear waves in combination with a cross-shore current. The mean (wave-averaged) velocities and related quantities (bottom roughness and water depth) and the mean mass transport (wave-averaged Lagrangian velocity) within the seabed boundary layer have been investigated for a range of wave conditions plus the cross-shore current. All analyses consistently indicate that the slope is to reduce streaming-induced mean velocities at a given water depth. It is observed that the streaming-induced mean velocities increase as the water depth decreases. As expected the boundary layer thickness and the maximum streaming velocity increase as the bottom roughness increases.

Introduction

Models for predicting waves propagating obliquely from deep to shallow water over a uniform gentle slope with the presence of a cross-shore current are important to understanding the underlying process of the near-shore seabed profiles in coastal environments. The first analytical solution for waves over a uniform gentle slope was that obtained by Biesel [2]. He suggested to approximate the normal incident waves propagating on a sloping plane by considering the bottom slope as a perturbation parameter in the velocity potential. With recent development in the numerical model, Chen et al. [4] used a perturbation method to derive the expression of the velocity potential in terms of bottom slope parameter α to second order for a progressive wave propagating obliquely over a gentle plane slope; their first order solution reduces to that of Biesel's [2] for the normal incident waves propagating on a sloping bottom. However, the boundary layer streaming effects were not accounted for.

Steady streaming in near-bed ocean flows is caused by both wave asymmetry (i.e. by asymmetry of turbulent fluctuations in successive wave half-cycles) as explained by Scandura [7] and by the presence of a small vertical wave velocity as explained by Longuet-Higgins [6]. The mild bottom slope modifies these mechanisms and is the focus of this study. This work presents a comparative study of linear waves propagating obliquely over a uniform gentle slope with a cross-shore current. The boundary layer equations have been solved using a central finite difference method in conjunction with a time-advancement scheme. Numerical simulations of the resulting seabed boundary layer flow are presented using Biesel's solution as the free stream velocity driving the seabed boundary layer. The effects of bottom roughness and water depth on the boundary layer flow are investigated for different angles of the sloping bed and different angles between the propagating waves and the cross-shore current, respectively.

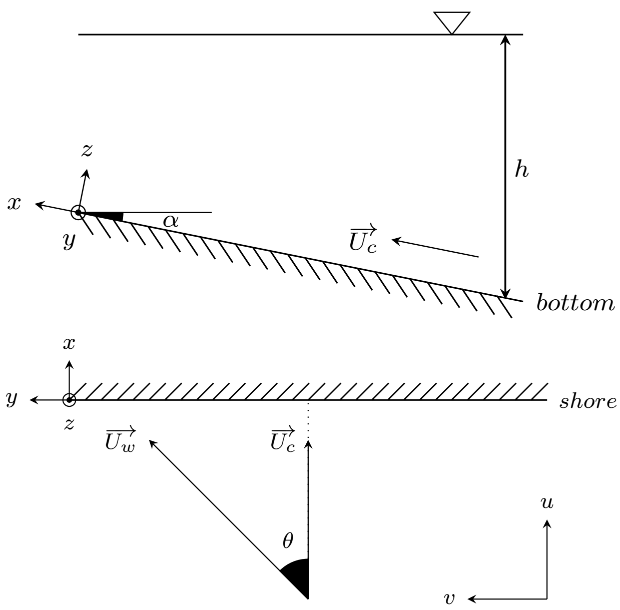


Figure 1: Definition sketch for waves propagating at an angle θ relative to the cross-shore current over a uniform gentle slope α. Here h is the water depth, $\overrightarrow{U_w}$ denotes the waves, $\overrightarrow{U_c}$ denotes the cross-shore current, and $|\overrightarrow{U_c}| = U_c$.

Model Formulation

For wave-dominated flows the wave-induced velocity component along the sloping bottom in the near-bed region is much larger than the wave-induced velocity component normal to the bottom, and hence the boundary layer approximation applies. The boundary layer equations have been solved using a central finite difference method in conjunction with a time-advancement integrator VODE [3]. A Cartesian coordinate system is defined such that the horizontal coordinate x is shorewards along the sloping bottom, y is in the long-shore direction, and z is the normal distance from the sloping bottom (in the right-hand x-y system) as shown in Figure 1. The bottom is fixed at $z = z_0 = k_N/30$, where k_N is the equivalent Nikuradse roughness for sand. In order to simplify the equations the relations

$$\frac{\partial \phi}{\partial x} = -\frac{1}{c_p}\frac{\cos\theta}{\cos\alpha}\frac{\partial \phi}{\partial t} \quad (1)$$

$$\frac{\partial \phi}{\partial y} = -\frac{\sin\theta}{c_p}\frac{\partial \phi}{\partial t} \quad (2)$$

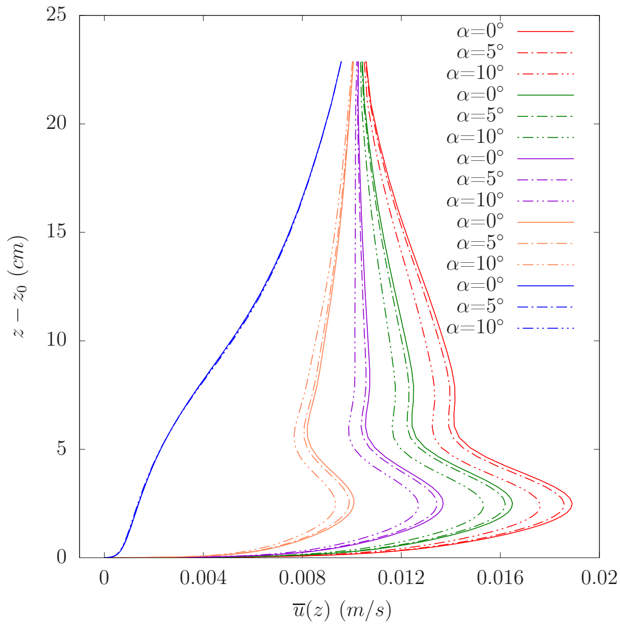
are applied; here ϕ is a boundary layer flow quantity beneath linear waves forcing, c_p is the wave celerity, α is the sea bottom slope and θ is the angle between the waves and the cross-shore current as shown in Figure 1. These relations reduce the three-dimensional boundary layer equations to spatially one-dimensional equations. A standard high Reynolds number k-ε model has been adopted to provide the turbulence closure. Geometric stretching of the grid is implemented to obtain a fine resolution close to the bed. A resolution of 100 vertical grid cells is found to be sufficient for resolving the boundary layer with a reasonable degree of accuracy. Dirichlet conditions are used for the velocity on the top of the boundary layer; the logarithmic wall law for rough turbulent flow is applied at the bottom. An equivalent wave model has been applied to represent the random waves. Moreover, the dispersion relation approximation for waves alone has been applied, neglecting the effect of the cross-shore current on the wave number (due to wave-dominated flow conditions) [5]. Furthermore, the near-bed free stream velocity driving the boundary layer is given by Chen et al. [4] considering the first order solution which accounts for both the effect of wave steepness and bottom slope. It should be noted that for normal incident waves, these velocities reduce to Biesel's [2] solution.

Results and Discussion

The flow within the seabed boundary layer over a uniform gently sloping for realistic wave-dominated conditions, bed roughnesses and water depth is presented. Ocean surface waves with an amplitude of $a = 1.22$ m and a period of 6 s propagate over a flat rough sloping bottom. At a given horizontal location, the water depth is 12 m, the wave length is 50 m, and the bottom roughnesses are $z_0 = 6.5 \cdot 10^{-6}$ m and $6.5 \cdot 10^{-5}$ m, corresponding to $A/k_{\mathrm{N}} = 3000$ and 300, respectively. If the empirical formula $k_{\mathrm{N}} = 2.5d_{50}$ is applied, these roughnesses correspond to very fine silt and very fine sand, respectively [8]. It should be noted that z_0 represents a given roughness of a uniform gentle slope. The given wave condition represents intermediate water depth ($k_p h = 1.48$) with wave steepness $ak_p = 0.151$. The cross-shore current is specified as $U_c = 0.01$ m/s at $z_{\max} = 0.25$ m above the sloping bottom. A sketch of the flow is given in Figure 1. The angle θ between the cross-shore current and the wave propagation direction varies from 0° to 90°; the results for the corresponding angles between 90° and 180° follow by symmetry.

Figure 2 shows the cross-shore $\overline{u}(z)$ and long-shore $\overline{v}(z)$ mean boundary layer velocity profiles beneath waves propagating at $\theta = 0°$, 30°, 45°, 60° and 90° for $A/k_{\mathrm{N}} = 300$ with the cross-shore current $U_c = 0.01$ m/s over a uniform gently sloping bottom where α is 5° and 10°. The mean velocity profiles for a flat bottom ($\alpha = 0°$) are given for comparison. It is observed that for a given angle θ the mean cross-shore velocity profile $\overline{u}(z)$ and long-shore velocity profile $\overline{v}(z)$ decrease as α increases. Thus the effect of the slope is to reduce the streaming-induced velocity; this reduction is larger for $\alpha = 10°$ than for $\alpha = 5°$, and both the velocity profiles are smaller than that for the flat bed. It should be noted that for the given cross-shore current the mean velocity components decrease in the current direction and increase normal to the current as the angle between the waves and the current increases, as previously found by Afzal et al. [1] for following waves and current where the wave propagation forms a nonzero angle with the current.

Figure 3 shows the effect of the bottom roughness on the mean velocity profiles beneath waves propagating normal to the shore for $A/k_{\mathrm{N}} = 300$ and 3000 with the cross-shore current $U_c = 0.01$ m/s over a uniform gently sloping bottom $\alpha = 5°$



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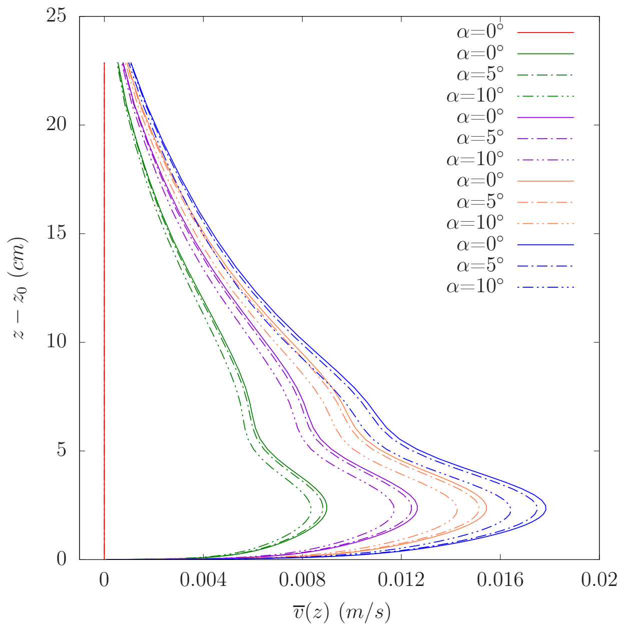

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Figure 2: The cross-shore $\overline{u}(z)$ and long-shore $\overline{v}(z)$ mean boundary layer velocity profiles beneath waves propagating at an angle θ for $A/k_{\mathrm{N}} = 300$ with the cross-shore current $U_c = 0.01$ m/s over a uniform gentle slope α. Lines denote: red $\theta = 0°$; green $\theta = 30°$; purple $\theta = 45°$; orange $\theta = 60°$; blue $\theta = 90°$.

and $\alpha = 10°$. The corresponding mean velocity profiles for a flat bottom ($\alpha = 0°$) are given for comparison. It is observed that the magnitudes of the streaming-induced velocities decrease as α increases; the effect of the slope is to reduce the streaming-induced velocities. As expected both the boundary layer thickness and the maximum streaming velocity increase as the bottom roughness increases i.e. from $A/k_{\mathrm{N}} = 3000$ to 300 corresponding to $z_0 = 6.5 \cdot 10^{-6}$ m and $6.5 \cdot 10^{-5}$ m, respectively.

Figure 4 shows the effect of the water depth (i.e. horizontal location) on the mean velocity profiles for waves propagating normal to the shore for the bottom roughness $z_0 = 6.5 \cdot 10^{-5}$ m with the cross-shore current U_c = 0.01 m/s where α is 5°

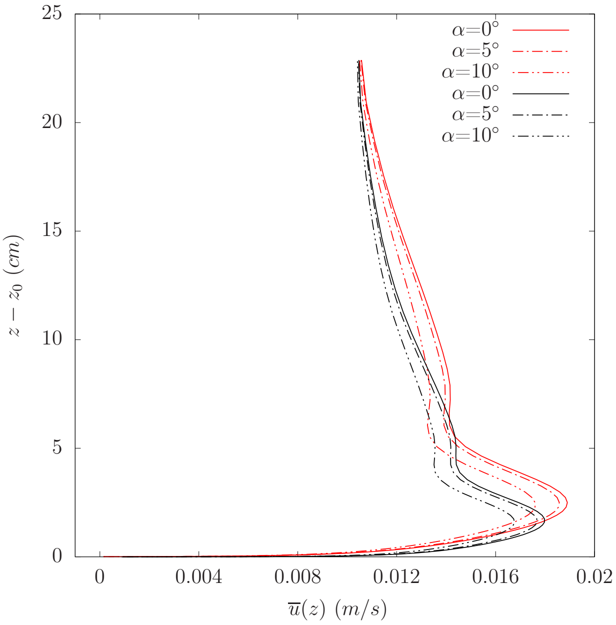


Figure 3: The cross-shore $\overline{u}(z)$ and long-shore $\overline{v}(z)$ mean boundary layer velocity profiles beneath waves propagating normal to the shore for two different bed roughnesses with the cross-shore current U_c = 0.01 m/s over a uniform gentle slope α. Lines denote: red $A/k_N = 300$; black $A/k_N = 3000$.

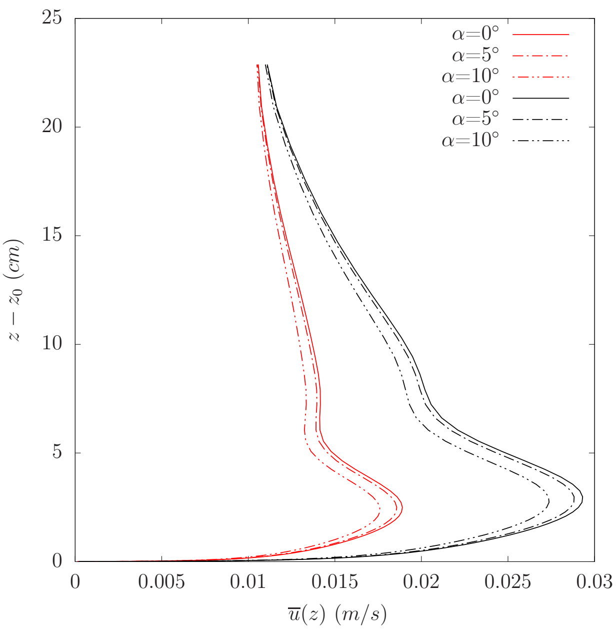


Figure 4: The cross-shore $\overline{u}(z)$ and long-shore $\overline{v}(z)$ mean boundary layer velocity profiles beneath waves propagating normal to the shore with the cross-shore current U_c = 0.01 m/s over a uniform gentle slope α. Lines denote: red h = 12 m; black h = 10 m.

and 10°. The corresponding mean velocity profiles for a flat bottom ($\alpha = 0°$) are given for comparison. It is observed that the streaming-induced mean velocities increase as the water depth decreases from $h = 12$ m to 10 m. This increase is due to the increase of the wave steepness; $ak_p = 0.151$ and 0.158 for $h = 12$ m and 10 m, respectively. Large wave steepness leads to larger wave-action and thus a larger streaming induced velocity. Moreover, the effect of the slope is to reduce the mean velocities for each horizontal location; this reduction of the mean velocities at a given horizontal location is larger for $\alpha = 10°$ than for $\alpha = 5°$, and both the velocity profiles are smaller than that for the flat bed.

The mean mass transport velocity (i.e. the wave-averaged Lagrangian velocity) is of particular importance in near-coastal waters, to provide the information of the spreading of seabed material e.g. including sediments, chemical compounds, and biological material such as fish larvae and phytoplankton for shallow waters. Figure 5 shows the cross-shore $\overline{u_L}(z)$ and long-shore $\overline{v_L}(z)$ mean mass transport velocity profiles beneath waves propagating at $\theta = 0°$, 30°, 45°, 60° and 90° for $A/k_N = 300$ with the cross-shore current U_c = 0.01 m/s over a uniform gentle slope where α is 5° and 10°. The mean mass transport velocity profiles for a flat bottom ($\alpha = 0°$) are given for comparison. The cross-shore and long-shore mean mass transport velocity profiles follow similar behaviour as the corresponding cross-shore and long-shore mean velocity profiles shown in Figure 2. It is observed that for a given angle θ, $\overline{u_L}(z)$ and $\overline{v_L}(z)$ decrease as α increases; the effect of the slope is to reduce the cross-shore and long-shore mass transport velocities. It is also noted that for the given cross-shore current the mass transport velocity components decrease in the current direction and increase normal to the current as the angle between the waves and the current increases, as previously found by Afzal et al. [1] for following waves and current where the wave propagation forms a nonzero angle with the current.

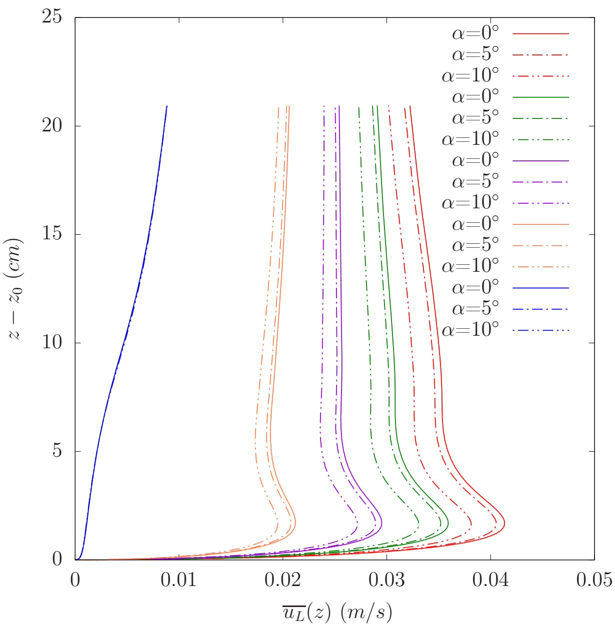
Conclusions

This work presents a comparative study of linear waves propagating obliquely over a uniform gentle slope in combination with a cross-shore current. The effect of the bottom slope on the mean (wave-averaged) velocities and related quantities (bottom roughness and water depth) have been investigated for realistic physical situations. Moreover, the mass transport (wave-averaged Lagrangian velocity) has been studied for a range of wave conditions over the uniform gentle slope. The main conclusions from this study can be summarized as follows:

- The streaming-induced velocities beneath obliquely propagating linear waves plus the cross-shore current are reduced as the angle of the uniform gentle slope increases. As the angle between the waves and the cross-shore current increases, the mean velocity components decrease in the current direction and increase normal to the current.
- As expected the boundary layer thickness and the maximum streaming velocity increase as the bottom roughness increases.
- The streaming-induced velocities increase as the water depth decreases and the effect of the slope is to reduce the mean velocity at a given horizontal location.
- The cross-shore and long-shore mean mass transport velocity profiles (i.e. the wave-averaged Lagrangian velocity) share similar features as the corresponding cross-shore and long-shore mean velocity profiles. As the angle

between the waves and the cross-shore current increases, the mean mass transport velocity components decrease in the current direction and increase normal to the current. The effect of the slope is to reduce the mean mass transport velocity.

- Overall, the reduction of the mean velocities due to the uniform gentle slope is larger for $\alpha = 10°$ than for $\alpha = 5°$, and both the velocity profiles are smaller than that for the flat bed.



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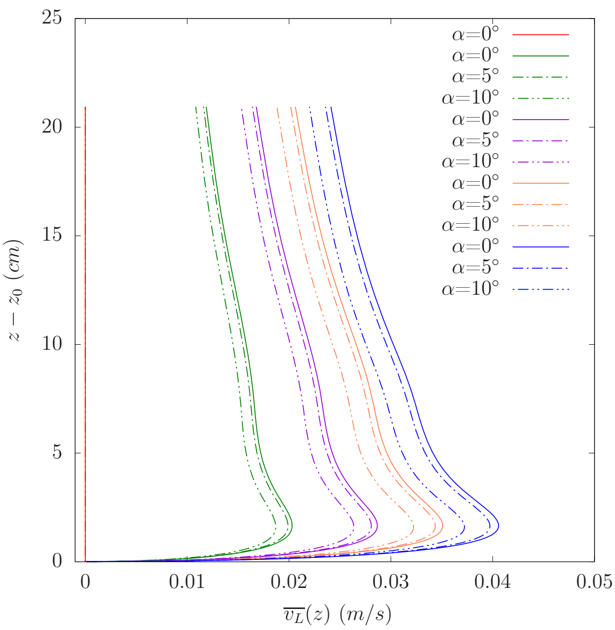

(b)

Figure 5: The cross-shore $\overline{u_L}(z)$ and long-shore $\overline{v_L}(z)$ mean mass transport velocity profiles beneath waves propagating at an angle θ for $A/k_N = 300$ with the cross-shore current $U_c = 0.01$ m/s over a uniform gentle slope α. Lines denote: red $\theta = 0°$; green $\theta = 30°$; purple $\theta = 45°$; orange $\theta = 60°$; blue $\theta = 90°$.

Acknowledgements

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